# 4次方程式における4重根の精度 (非線型方程式の数値解析)



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# 4次方程式における4重根の精度

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$$x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0 = 0 \qquad (1)$$

解は、$x_1 = x_2 = x_3 = x_4 = \alpha \cdot 10^{\beta}$ である。

解と係数の関係より、次式が得られる。

$$\begin{cases} a_3 = -4\alpha \cdot 10^{\beta} + [10^{\beta-m}] \\ a_2 = 6\alpha^2 \cdot 10^{2\beta} + [10^{2\beta-m}] \\ a_1 = -4\alpha^3 \cdot 10^{3\beta} + [10^{3\beta-m}] \\ a_0 = \alpha^4 \cdot 10^{4\beta} + [10^{4\beta-m}] \end{cases}$$

[ ] 内の桁以下は計算に用いられない桁

$m$ : 計算桁数

$x = y + \bar{x}$ を用いて、4次方程式 (1) を3次の係数が零である4次方程式 (2) に変換する。

$$y^4 + k'y^2 + l'y + m' = 0 \qquad (2)$$

4次方程式 (2) の係数は、次のように、誤差のみになる。

$$\begin{cases} \bar{x} = -\frac{1}{4} a_3 = \alpha \cdot 10^{\beta} + [10^{\beta-m}] \\ k' = -\frac{3}{8} a_3^2 + a_2 = -6\alpha^2 \cdot 10^{2\beta} + 6\alpha^2 \cdot 10^{2\beta} + [10^{2\beta-m}] \\ \quad = \varepsilon_{k'} \cdot 10^{2\beta-m} + [10^{2\beta-2m}] \end{cases}$$

$$\begin{cases} l' = \frac{1}{8}a_3^3 - \frac{1}{2}a_3 a_2 + a_1 \\ \quad = -8\alpha^3 \cdot 10^{3\beta} + 12\alpha^3 \cdot 10^{3\beta} - 4\alpha^3 \cdot 10^{3\beta} + [10^{3\beta-m}] \\ \quad = \varepsilon_{l'} \cdot 10^{3\beta-m} + [10^{3\beta-2m}] \\ m' = -\frac{3}{4^4}a_3^4 + \frac{1}{4^2}a_3^2 a_2 - \frac{1}{4}a_3 a_1 + a_0 \\ \quad = -3\alpha^4 \cdot 10^{4\beta} + 6\alpha^4 \cdot 10^{4\beta} - 4\alpha^4 \cdot 10^{4\beta} + \alpha^4 \cdot 10^{4\beta} + [10^{4\beta-m}] \\ \quad = \varepsilon_{m'} \cdot 10^{4\beta-m} + [10^{4\beta-2m}] \end{cases}$$

4次方程式(2)が4次方程式 $(y^2+u)^2-(vy+w)^2=0$ に変形できるように、$u$, $v$, $w$ を求める。

$$\begin{cases} v^2 = 2u - k' = 2u - \varepsilon_{k'} \cdot 10^{2\beta-m} + [10^{2\beta-2m}] \\ v^2 w^2 = l'^2/4 \quad = \frac{1}{4}\varepsilon_{l'}^2 \cdot 10^{6\beta-2m} + [10^{6\beta-3m}] \\ w^2 = u^2 - m' = u^2 - \varepsilon_{m'} \cdot 10^{4\beta-m} + [10^{4\beta-2m}] \end{cases} \tag{3}$$

$$v^2 w^2 = (2u-k')(u^2-m') = l'^2/4$$

$$(2u - \varepsilon_{k'} \cdot 10^{2\beta-m} + [10^{2\beta-2m}])(u^2 - \varepsilon_{m'} \cdot 10^{4\beta-m} + [10^{4\beta-2m}])$$
$$= \frac{1}{4}\varepsilon_{l'}^2 \cdot 10^{6\beta-2m} + [10^{6\beta-3m}] \tag{4}$$

$$u^3 + \left\{-\frac{\varepsilon_{k'}}{2} \cdot 10^{2\beta-m} + [10^{2\beta-2m}]\right\}u^2 + \left\{-\varepsilon_{m'} \cdot 10^{4\beta-m} + [10^{4\beta-2m}]\right\}u$$
$$+ \left\{\left(\frac{\varepsilon_{k'} \cdot \varepsilon_{m'}}{2} - \frac{\varepsilon_{l'}^2}{8}\right) \cdot 10^{6\beta-2m} + [10^{6\beta-3m}]\right\} = 0 \tag{5}$$

座標変換(6)により、(5)を2次の係数が零である3次方程式(7)に変換する。

$$u = z + \left\{\frac{\varepsilon_{k'}}{6} \cdot 10^{2\beta-m} + [10^{2\beta-2m}]\right\} \tag{6}$$

$$z^3 + 3pz + q = 0 \tag{7}$$

$$\begin{cases} p = -\dfrac{\varepsilon_{m'}}{3}\cdot 10^{4\beta-m} + [\,10^{4\beta-2m}\,] \\[2ex] q = \left(\dfrac{\varepsilon_{k'}\cdot\varepsilon_{m'}}{3} - \dfrac{\varepsilon_{l'}^2}{8}\right)\cdot 10^{6\beta-2m} + [\,10^{6\beta-3m}\,] \end{cases}$$

$$q^2 + 4p^3 = -\frac{4}{27}\,\varepsilon_{m'}^2\cdot 10^{12\beta-3m} + [\,10^{12\beta-4m}\,]$$

$$\sqrt{q^2+4p^3} = 2\sqrt{\frac{\varepsilon_{m'}^3}{27}}\; i\cdot 10^{6\beta-3m/2} + [\,10^{6\beta-5m/2}\,]$$

$$\left(\sqrt{q^2+4p^3} - q\right)/2 = \sqrt{\frac{\varepsilon_{m'}^3}{27}}\; i\cdot 10^{6\beta-3m/2} + \left(\frac{\varepsilon_{l'}^2}{16} - \frac{\varepsilon_{k'}\cdot\varepsilon_{m'}}{6}\right)\cdot 10^{6\beta-2m}$$
$$+ [\,10^{6\beta-5m/2}\,]$$

$$\left(-\sqrt{q^2+4p^3} - q\right)/2 = -\sqrt{\frac{\varepsilon_{m'}^3}{27}}\; i\cdot 10^{6\beta-3m/2}$$
$$+\left(\frac{\varepsilon_{l'}^2}{16} - \frac{\varepsilon_{k'}\cdot\varepsilon_{m'}}{6}\right)\cdot 10^{6\beta-2m} + [\,10^{6\beta-5m/2}\,]$$

$$s = \sqrt[3]{\left(\sqrt{q^2+4p^3} - q\right)/2}$$
$$= \frac{(\sqrt{3}+i)}{2}\cdot\sqrt{\frac{\varepsilon_{m'}}{3}}\cdot 10^{2\beta-m/2}\left\{1 - \frac{1}{3}\sqrt{\frac{27}{\varepsilon_{m'}^3}}\left(\frac{\varepsilon_{l'}^2}{16} - \frac{\varepsilon_{k'}\cdot\varepsilon_{m'}}{6}\right) i\cdot 10^{-m/2} + [\,10^{-m}\,]\right\}$$
$$= \frac{(\sqrt{3}+i)}{2}\cdot\sqrt{\frac{\varepsilon_{m'}}{3}}\cdot 10^{2\beta-m/2} + \frac{(1-\sqrt{3}\,i)}{2\,\varepsilon_{m'}}\left(\frac{\varepsilon_{l'}^2}{16} - \frac{\varepsilon_{k'}\cdot\varepsilon_{m'}}{6}\right)\cdot 10^{2\beta-m}$$
$$+ [\,10^{2\beta-3m/2}\,]$$

$$t = \sqrt[3]{\left(-\sqrt{q^2+4p^3} - q\right)/2}$$
$$= \frac{(\sqrt{3}-i)}{2}\cdot\sqrt{\frac{\varepsilon_{m'}}{3}}\cdot 10^{2\beta-m/2}\left\{1 + \frac{1}{3}\sqrt{\frac{27}{\varepsilon_{m'}^3}}\left(\frac{\varepsilon_{l'}^2}{16} - \frac{\varepsilon_{k'}\cdot\varepsilon_{m'}}{6}\right) i\cdot 10^{-m/2} + [\,10^{-m}\,]\right\}$$

$$=\frac{(\sqrt{3}-i)}{2}\cdot\sqrt{\frac{\varepsilon_m'}{3}}\cdot 10^{2\beta-m/2}+\frac{(1+\sqrt{3}\,i)}{2\,\varepsilon_m'}\left(\frac{\varepsilon_\ell'^2}{16}-\frac{\varepsilon_k'\cdot\varepsilon_m'}{6}\right)\cdot 10^{2\beta-m}$$

$$+[10^{2\beta-3m/2}]$$

$$\omega\cdot s=\frac{(-\sqrt{3}+i)}{2}\cdot\sqrt{\frac{\varepsilon_m'}{3}}\cdot 10^{2\beta-m/2}+\frac{(1+\sqrt{3}\,i)}{2\,\varepsilon_m'}\left(\frac{\varepsilon_\ell'^2}{16}-\frac{\varepsilon_k'\cdot\varepsilon_m'}{6}\right)\cdot 10^{2\beta-m}$$

$$+[10^{2\beta-3m/2}] \qquad (\omega^3=1.0)$$

$$\omega\cdot t=i\cdot\sqrt{\frac{\varepsilon_m'}{3}}\cdot 10^{2\beta-m/2}-\frac{1}{\varepsilon_m'}\left(\frac{\varepsilon_\ell'^2}{16}-\frac{\varepsilon_k'\cdot\varepsilon_m'}{6}\right)\cdot 10^{2\beta-m}+[10^{2\beta-3m/2}]$$

$$\omega^2\cdot s=-i\cdot\sqrt{\frac{\varepsilon_m'}{3}}\cdot 10^{2\beta-m/2}-\frac{1}{\varepsilon_m'}\left(\frac{\varepsilon_\ell'^2}{16}-\frac{\varepsilon_k'\cdot\varepsilon_m'}{6}\right)\cdot 10^{2\beta-m}+[10^{2\beta-3m/2}]$$

$$\omega^2\cdot t=-\frac{(\sqrt{3}+i)}{2}\cdot\sqrt{\frac{\varepsilon_m'}{3}}\cdot 10^{2\beta-m/2}+\frac{(1-\sqrt{3}\,i)}{2\,\varepsilon_m'}\left(\frac{\varepsilon_\ell'^2}{16}-\frac{\varepsilon_k'\cdot\varepsilon_m'}{6}\right)\cdot 10^{2\beta-m}$$

$$+[10^{2\beta-3m/2}]$$

したがって．(7) の解 $Z_1$, $Z_2$, $Z_3$ は (8) である．

$$\begin{cases} Z_1=s+t \\ \quad=\sqrt{\varepsilon_m'}\cdot 10^{2\beta-m/2}+\dfrac{1}{\varepsilon_m'}\left(\dfrac{\varepsilon_\ell'^2}{16}-\dfrac{\varepsilon_k'\cdot\varepsilon_m'}{6}\right)\cdot 10^{2\beta-m}+[10^{2\beta-3m/2}] \\ Z_2=\omega s+\omega^2 t \\ \quad=-\sqrt{\varepsilon_m'}\cdot 10^{2\beta-m/2}+\dfrac{1}{\varepsilon_m'}\left(\dfrac{\varepsilon_\ell'^2}{16}-\dfrac{\varepsilon_k'\cdot\varepsilon_m'}{6}\right)\cdot 10^{2\beta-m}+[10^{2\beta-3m/2}] \\ Z_3=\dfrac{-q}{Z_1\cdot Z_2}=\quad-\dfrac{1}{\varepsilon_m'}\left(\dfrac{\varepsilon_\ell'^2}{8}-\dfrac{\varepsilon_k'\cdot\varepsilon_m'}{3}\right)\cdot 10^{2\beta-m}+[10^{2\beta-2m}] \end{cases} \qquad (8)$$

$$\{\ Z_1 \cdot Z_2 = (s+t)(\omega s + \omega^2 t) = -\varepsilon_{m'} \cdot 10^{4\beta - m} + [\,10^{4\beta - 2m}\,]\ \}$$

$Z_3$ を $\omega^2 s + \omega t$ より計算すると桁落ち誤差が入るので、$Z_3$ は解と係数の関係より求める。(8) を (6) に代入して、3次方程式 (5) の解 $u_1$, $u_2$, $u_3$ (9) を求める。

$$\begin{cases} u_1 = \sqrt{\varepsilon_{m'}} \cdot 10^{2\beta - m/2} + \dfrac{\varepsilon_{l'}^2}{16\,\varepsilon_{m'}} \cdot 10^{2\beta - m} + [\,10^{2\beta - 3m/2}\,] \\ u_2 = -\sqrt{\varepsilon_{m'}} \cdot 10^{2\beta - m/2} + \dfrac{\varepsilon_{l'}^2}{16\,\varepsilon_{m'}} \cdot 10^{2\beta - m} + [\,10^{2\beta - 3m/2}\,] \\ u_3 = \left(\dfrac{\varepsilon_{k'}}{2} - \dfrac{\varepsilon_{l'}^2}{8\,\varepsilon_{m'}}\right) \cdot 10^{2\beta - m} + [\,10^{2\beta - 2m}\,] \end{cases} \qquad (9)$$

$u_1$ (9) を用いて (2) を因数分解した2つの2次方程式 (12), (13) の係数を求める。

$$2u_1 - k' = 2\sqrt{\varepsilon_{m'}} \cdot 10^{2\beta - m/2} + \left(\frac{\varepsilon_{l'}^2}{8\,\varepsilon_{m'}} - \varepsilon_{k'}\right) \cdot 10^{2\beta - m} + [\,10^{2\beta - 3m/2}\,]$$

$$v_1 = \sqrt{2u_1 - k'}$$

$$= \sqrt{2} \cdot \sqrt[4]{\varepsilon_{m'}} \cdot 10^{\beta - m/4} + \frac{1}{2 \cdot \sqrt{2} \cdot \sqrt[4]{\varepsilon_{m'}}} \left(\frac{\varepsilon_{l'}^2}{8\,\varepsilon_{m'}} - \varepsilon_{k'}\right) \cdot 10^{\beta - 3m/4} + [\,10^{\beta - 5m/4}\,] \qquad (10)$$

$$u_1^2 = \varepsilon_{m'} \cdot 10^{4\beta - m} + \frac{\varepsilon_{l'}^2}{8\sqrt{\varepsilon_{m'}}} \cdot 10^{4\beta - 3m/2} + [\,10^{4\beta - 2m}\,]$$

$w_1$ を $\sqrt{u_1^2 - m'}$ より計算すると桁落ち誤差が入るので、(11) で求める。

$$w_1=-\frac{l'}{2v_1}=-\frac{\varepsilon_{l'}}{2\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}}\cdot 10^{2\beta-3m/4}$$

$$-\frac{\varepsilon_{l'}}{8\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}\cdot\sqrt{\varepsilon_{m'}}}\left(\frac{\varepsilon_{l'}^2}{8\varepsilon_{m'}}-\varepsilon_{k'}\right)\cdot 10^{2\beta-5m/4}+[10^{2\beta-7m/4}]$$

(11)

$$y^2+v_1y+(u_1+w_1)=0 \qquad (12)$$

$$y^2-v_1y+(u_1-w_1)=0 \qquad (13)$$

2次方程式 (12) より (2) の解 $y_1$, $y_2$ を求める。

$$v_1=\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}\cdot 10^{\beta-m/4}+\frac{1}{2\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}}\left(\frac{\varepsilon_{l'}^2}{8\varepsilon_{m'}}-\varepsilon_{k'}\right)\cdot 10^{\beta-3m/4}+[10^{\beta-5m/4}]$$

$$u_1+w_1=\sqrt{\varepsilon_{m'}}\cdot 10^{2\beta-m/2}-\frac{\varepsilon_{l'}}{2\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}}\cdot 10^{2\beta-3m/4}+\frac{\varepsilon_{l'}^2}{16\varepsilon_{m'}}\cdot 10^{2\beta-m}$$

$$+\frac{\varepsilon_{l'}}{8\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}\cdot\sqrt{\varepsilon_{m'}}}\left(\frac{\varepsilon_{l'}}{8\varepsilon_{m'}}-\varepsilon_{k'}\right)\cdot 10^{2\beta-5m/4}+[10^{2\beta-3m/2}]$$

$$v_1^2=\ 2\sqrt{\varepsilon_{m'}}\cdot 10^{2\beta-m/2}\qquad\qquad+\left(\frac{\varepsilon_{l'}^2}{8\varepsilon_{m'}}-\varepsilon_{k'}\right)\cdot 10^{2\beta-m}+[10^{2\beta-3m/2}]$$

$$-4(u_1+w_1)=-4\sqrt{\varepsilon_{m'}}\cdot 10^{2\beta-m/2}+\frac{\sqrt{2}\cdot\varepsilon_{l'}}{\sqrt[4]{\varepsilon_{m'}}}\cdot 10^{2\beta-3m/4}-\frac{\varepsilon_{l'}^2}{4\varepsilon_{m'}}\cdot 10^{2\beta-m}$$

$$-\frac{\varepsilon_{l}}{2\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}\sqrt{\varepsilon_{m'}}}\left(\frac{\varepsilon_{l'}^2}{8\varepsilon_{m'}}-\varepsilon_{k'}\right)\cdot 10^{2\beta-5m/4}+[10^{2\beta-3m/2}]$$

$$v_1^2-4(u_1+w_1)=-2\sqrt{\varepsilon_{m'}}\cdot 10^{2\beta-m/2}\left\{1-\frac{\varepsilon_{l'}}{\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}\cdot\sqrt{\varepsilon_{m'}}}\cdot 10^{-m/4}\right.$$

$$+\frac{1}{2\sqrt{\varepsilon_{m'}}}\left(\frac{\varepsilon_{l'}^2}{8\varepsilon_{m'}}+\varepsilon_{k'}\right)\cdot 10^{-m/2}$$

$$\left.+\frac{\varepsilon_{l'}}{4\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}\cdot\varepsilon_{m'}}\left(\frac{\varepsilon_{l'}^2}{8\varepsilon_{m'}}-\varepsilon_{k'}\right)\cdot 10^{-3m/4}+[10^{-m}]\right\}$$

$$\sqrt{v_1^2-4(u_1+w_1)} = \sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}\cdot i\cdot 10^{\beta-m/4} - \frac{\varepsilon_{l'}}{2\cdot\sqrt{\varepsilon_{m'}}}\cdot i\cdot 10^{\beta-m/2}$$
$$- \frac{1}{2\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}}\left(\frac{\varepsilon_{l'}^2}{8\,\varepsilon_{m'}}-\varepsilon_{k'}\right)\cdot i\cdot 10^{\beta-3m/4} + [10^{\beta-5m/4}]$$

$$\left\{\begin{aligned} y_1 &= \frac{1}{2}\left\{-v_1+\sqrt{v_1^2-4(u_1+w_1)}\right\} = -\frac{(1-i)}{\sqrt{2}}\cdot\sqrt[4]{\varepsilon_{m'}}\cdot 10^{\beta-m/4} \\ &\quad - \frac{\varepsilon_{l'}}{4\cdot\sqrt{\varepsilon_{m'}}}\cdot i\cdot 10^{\beta-m/2} - \frac{(1+i)}{4\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}}\left(\frac{\varepsilon_{l'}^2}{8\,\varepsilon_{m'}}-\varepsilon_{k'}\right)\cdot 10^{\beta-3m/4} + [10^{\beta-5m/4}] \\ y_2 &= \frac{1}{2}\left\{-v_1-\sqrt{v_1^2-4(u_1+w_1)}\right\} = -\frac{(1+i)}{\sqrt{2}}\cdot\sqrt[4]{\varepsilon_{m'}}\cdot 10^{\beta-m/4} \\ &\quad + \frac{\varepsilon_{l'}}{4\cdot\sqrt{\varepsilon_{m'}}}\cdot i\cdot 10^{\beta-m/2} - \frac{(1-i)}{4\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}}\left(\frac{\varepsilon_{l'}^2}{8\,\varepsilon_{m'}}-\varepsilon_{k'}\right)\cdot 10^{\beta-3m/4} + [10^{\beta-5m/4}] \end{aligned}\right. \tag{14}$$

2次方程式 (13) より (2) の解 $y_3$, $y_4$ を求める。

$$u_1-w_1 = \sqrt{\varepsilon_{m'}}\cdot 10^{2\beta-m/2} + \frac{\varepsilon_{l'}}{2\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}}\cdot 10^{2\beta-3m/4} + \frac{\varepsilon_{l'}^2}{16\,\varepsilon_{m'}}\cdot 10^{2\beta-m}$$
$$- \frac{\varepsilon_{l'}}{8\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}\cdot\sqrt{\varepsilon_{m'}}}\left(\frac{\varepsilon_{l'}^2}{8\,\varepsilon_{m'}}-\varepsilon_{k'}\right)\cdot 10^{2\beta-5m/4} + [10^{2\beta-3m/2}]$$

$$-4(u_1-w_1) = -4\sqrt{\varepsilon_{m'}}\cdot 10^{2\beta-m/2} - \frac{\sqrt{2}\cdot\varepsilon_{l'}}{\sqrt[4]{\varepsilon_{m'}}}\cdot 10^{2\beta-3m/4} - \frac{\varepsilon_{l'}^2}{4\,\varepsilon_{m'}}\cdot 10^{2\beta-m}$$
$$+ \frac{\varepsilon_{l'}}{2\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}\cdot\sqrt{\varepsilon_{m'}}}\left(\frac{\varepsilon_{l'}^2}{8\,\varepsilon_{m'}}-\varepsilon_{k'}\right)\cdot 10^{2\beta-5m/4} + [10^{2\beta-3m/2}]$$

$$v_1^2-4(u_1-w_1) = -2\sqrt{\varepsilon_{m'}}\cdot 10^{2\beta-m/2}\left\{1+\frac{\varepsilon_{l'}}{\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}\cdot\sqrt{\varepsilon_{m'}}}\cdot 10^{-m/4}\right.$$
$$+ \frac{1}{2\sqrt{\varepsilon_{m'}}}\left(\frac{\varepsilon_{l'}^2}{8\,\varepsilon_{m'}}+\varepsilon_{k'}\right)\cdot 10^{-m/2}$$
$$\left. - \frac{\varepsilon_{l'}}{4\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}\cdot\varepsilon_{m'}}\left(\frac{\varepsilon_{l'}^2}{8\,\varepsilon_{m'}}-\varepsilon_{k'}\right)\cdot 10^{-3m/4} + [10^{-m}]\right\}$$

$$\sqrt{v_1^2-4(u_1-w_1)}=\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}\cdot i\cdot 10^{\beta-m/4}+\frac{\varepsilon_{l'}}{2\cdot\sqrt{\varepsilon_{m'}}}\cdot i\cdot 10^{\beta-m/2}$$
$$-\frac{1}{2\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}}\left(\frac{\varepsilon_{l'}^2}{8\,\varepsilon_{m'}}-\varepsilon_{k'}\right)\cdot i\cdot 10^{\beta-3m/4}+[10^{\beta-5m/4}]$$

$$\begin{cases} y_3=\dfrac{1}{2}\left\{v_1+\sqrt{v_1^2-4(u_1-w_1)}\right\}=\dfrac{(1+i)}{\sqrt{2}}\cdot\sqrt[4]{\varepsilon_{m'}}\cdot 10^{\beta-m/4} \\ \quad +\dfrac{\varepsilon_{l'}}{4\cdot\sqrt{\varepsilon_{m'}}}\cdot i\cdot 10^{\beta-m/2}+\dfrac{(1-i)}{4\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}}\left(\dfrac{\varepsilon_{l'}^2}{8\,\varepsilon_{m'}}-\varepsilon_{k'}\right)\cdot 10^{\beta-3m/4}+[10^{\beta-5m/4}] \\ y_4=\dfrac{1}{2}\left\{v_1-\sqrt{v_1^2-4(u_1-w_1)}\right\}=\dfrac{(1-i)}{\sqrt{2}}\cdot\sqrt[4]{\varepsilon_{m'}}\cdot 10^{\beta-m/4} \\ \quad -\dfrac{\varepsilon_{l'}}{4\cdot\sqrt{\varepsilon_{m'}}}\cdot i\cdot 10^{\beta-m/2}+\dfrac{(1+i)}{4\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}}\left(\dfrac{\varepsilon_{l'}^2}{8\,\varepsilon_{m'}}-\varepsilon_{k'}\right)\cdot 10^{\beta-3m/4}+[10^{\beta-5m/4}] \end{cases} \tag{15}$$

$u_2$ (9) を用いて (2) を因数分解した2つの2次方程式 (18), (19) の係数を求める。

$$2u_2-k'=-2\sqrt{\varepsilon_{m'}}\cdot 10^{2\beta-m/2}+\left(\frac{\varepsilon_{l'}^2}{8\,\varepsilon_{m'}}-\varepsilon_{k'}\right)\cdot 10^{2\beta-m}+[10^{2\beta-3m/2}]$$

$$v_2=\sqrt{2u_2-k'}$$
$$=\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}\cdot i\cdot 10^{\beta-m/4}-\frac{1}{2\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}}\left(\frac{\varepsilon_{l'}^2}{8\,\varepsilon_{m'}}-\varepsilon_{k'}\right)\cdot i\cdot 10^{\beta-3m/4}$$
$$+[10^{\beta-5m/4}] \tag{16}$$

$$u_2^2=\varepsilon_{m'}\cdot 10^{4\beta-m}-\frac{\varepsilon_{l'}^2}{8\sqrt{\varepsilon_{m'}}}\cdot 10^{4\beta-3m/2}+[10^{4\beta-2m}]$$

$w_2$ を $\sqrt{u_2^2-m'}$ より計算すると桁落ち誤差が入るので、(17)

で求める.

$$w_2 = -\frac{l'}{2v_2} = \frac{\varepsilon_{l'}}{2\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}}\cdot i\cdot 10^{2\beta-3m/4}$$

$$+\frac{\varepsilon_{l'}}{8\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}\cdot\sqrt{\varepsilon_{m'}}}\left(\frac{\varepsilon_{l'}^2}{8\varepsilon_{m'}}-\varepsilon_{k'}\right)\cdot i\cdot 10^{2\beta-5m/4}+[10^{2\beta-7m/4}] \tag{17}$$

$$y^2+v_2y+(u_2+w_2)=0 \tag{18}$$

$$y^2-v_2y+(u_2-w_2)=0 \tag{19}$$

2次方程式(18)より(2)の解$y_1$, $y_2$を求める.

$$v_2=\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}\cdot i\cdot 10^{\beta-m/4}-\frac{1}{2\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}}\left(\frac{\varepsilon_{l'}^2}{8\varepsilon_{m'}}-\varepsilon_{k'}\right)\cdot i\cdot 10^{\beta-3m/4}$$

$$+[10^{\beta-5m/4}]$$

$$u_2+w_2=-\sqrt{\varepsilon_{m'}}\cdot 10^{2\beta-m/2}+\frac{\varepsilon_{l'}}{2\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}}\cdot i\cdot 10^{2\beta-3m/4}+\frac{\varepsilon_{l'}^2}{16\varepsilon_{m'}}\cdot 10^{2\beta-m}$$

$$+\frac{\varepsilon_{l}}{8\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}\cdot\sqrt{\varepsilon_{m'}}}\left(\frac{\varepsilon_{l'}^2}{8\varepsilon_{m'}}-\varepsilon_{k}\right)\cdot i\cdot 10^{2\beta-5m/4}+[10^{2\beta-3m/2}]$$

$$v_2^2=-2\cdot\sqrt{\varepsilon_{m'}}\cdot 10^{2\beta-m/2}\qquad+\left(\frac{\varepsilon_{l'}^2}{8\varepsilon_{m'}}-\varepsilon_{k'}\right)\cdot 10^{2\beta-m}+[10^{2\beta-3m/2}]$$

$$-4(u_2+w_2)=4\cdot\sqrt{\varepsilon_{m'}}\cdot 10^{2\beta-m/2}-\frac{\sqrt{2}\cdot\varepsilon_{l'}}{\sqrt[4]{\varepsilon_{m'}}}\cdot i\cdot 10^{2\beta-3m/4}-\frac{\varepsilon_{l'}^2}{4\varepsilon_{m'}}\cdot 10^{2\beta-m}$$

$$-\frac{\varepsilon_{l'}}{2\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}\cdot\sqrt{\varepsilon_{m'}}}\left(\frac{\varepsilon_{l'}^2}{8\varepsilon_{m'}}-\varepsilon_{k'}\right)\cdot i\cdot 10^{2\beta-5m/4}+[10^{2\beta-3m/2}]$$

$$v_2^2-4(u_2+w_2)=2\cdot\sqrt{\varepsilon_{m'}}\cdot 10^{2\beta-m/2}\left\{1-\frac{\varepsilon_{l'}}{\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}\cdot\sqrt{\varepsilon_{m'}}}\cdot i\cdot 10^{-m/4}\right.$$

$$-\frac{1}{2\sqrt{\varepsilon_{m'}}}\left(\frac{\varepsilon_{l'}^2}{8\varepsilon_{m'}}+\varepsilon_{k'}\right)\cdot 10^{-m/2}$$

$$-\frac{\varepsilon_{\ell'}}{4\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}\cdot\varepsilon_{m'}}\left(\frac{\varepsilon_{\ell'}^2}{8\varepsilon_{m'}}-\varepsilon_{k'}\right)\cdot i\cdot 10^{-3m/4}+[10^{-m}]\Big\}$$

$$\sqrt{v_2^2-4(u_2+w_2)}=\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}\cdot 10^{\beta-m/4}-\frac{\varepsilon_{\ell'}}{2\cdot\sqrt{\varepsilon_{m'}}}\cdot i\cdot 10^{\beta-m/2}$$

$$+\frac{1}{2\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}}\left(\frac{\varepsilon_{\ell'}^2}{8\varepsilon_{m'}}-\varepsilon_{k'}\right)\cdot 10^{\beta-3m/4}+[10^{\beta-5m/4}]$$

$$\left\{\begin{aligned}
y_4&=\frac{1}{2}\left\{-v_2+\sqrt{v_2^2-4(u_2+w_2)}\right\}=\frac{(1-i)}{\sqrt{2}}\cdot\sqrt[4]{\varepsilon_{m'}}\cdot 10^{\beta-m/4}\\
&\quad-\frac{\varepsilon_{\ell'}}{4\cdot\sqrt{\varepsilon_{m'}}}\cdot i\cdot 10^{\beta-m/2}+\frac{(1+i)}{4\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}}\left(\frac{\varepsilon_{\ell'}^2}{8\varepsilon_{m'}}-\varepsilon_{k'}\right)\cdot 10^{\beta-3m/4}+[10^{\beta-5m/4}]\\
y_2&=\frac{1}{2}\left\{-v_2-\sqrt{v_2^2-4(u_2+w_2)}\right\}=-\frac{(1+i)}{\sqrt{2}}\cdot\sqrt[4]{\varepsilon_{m'}}\cdot 10^{\beta-m/4}\\
&\quad+\frac{\varepsilon_{\ell}}{4\cdot\sqrt{\varepsilon_{m'}}}\cdot i\cdot 10^{\beta-m/2}-\frac{(1-i)}{4\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}}\left(\frac{\varepsilon_{\ell'}^2}{8\varepsilon_{m'}}-\varepsilon_{k'}\right)\cdot 10^{\beta-3m/4}+[10^{\beta-5m/4}]
\end{aligned}\right. \tag{20}$$

2次方程式 (19) より (2) の解 $y_3$, $y_1$ を求める.

$$u_2-w_2=-\sqrt{\varepsilon_{m'}}\cdot 10^{2\beta-m/2}-\frac{\varepsilon_{\ell'}}{2\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}}\cdot i\cdot 10^{2\beta-3m/4}+\frac{\varepsilon_{\ell'}^2}{16\varepsilon_{m'}}\,10^{2\beta-m}$$

$$-\frac{\varepsilon_{\ell'}}{8\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}\sqrt{\varepsilon_{m'}}}\left(\frac{\varepsilon_{\ell'}^2}{8\varepsilon_{m'}}-\varepsilon_{k'}\right)\cdot i\cdot 10^{2\beta-5m/4}+[10^{2\beta-3m/2}]$$

$$-4(u_2-w_2)=4\sqrt{\varepsilon_{m'}}\cdot 10^{2\beta-m/2}+\frac{\sqrt{2}\cdot\varepsilon_{\ell'}}{\sqrt[4]{\varepsilon_{m'}}}\cdot i\cdot 10^{2\beta-3m/4}-\frac{\varepsilon_{\ell'}^2}{4\varepsilon_{m'}}\,10^{2\beta-m}$$

$$+\frac{\varepsilon_{\ell'}}{2\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}\cdot\sqrt{\varepsilon_{m'}}}\left(\frac{\varepsilon_{\ell'}^2}{8\varepsilon_{m'}}-\varepsilon_{k'}\right)\cdot i\cdot 10^{2\beta-5m/4}+[10^{2\beta-3m/2}]$$

$$v_2^2-4(u_2-w_2)=2\cdot\sqrt{\varepsilon_{m'}}\cdot 10^{2\beta-m/2}\left\{1+\frac{\varepsilon_{\ell'}}{\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}\cdot\sqrt{\varepsilon_{m'}}}\cdot i\cdot 10^{-m/4}\right.$$

$$-\frac{1}{2\cdot\sqrt{\varepsilon_{m'}}}\left(\frac{\varepsilon_{\ell'}^2}{8\varepsilon_{m'}}+\varepsilon_{k'}\right)\cdot 10^{-m/2}$$

$$+\frac{\varepsilon_{l'}}{4\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}\cdot\varepsilon_{m'}}\left(\frac{\varepsilon_{l'}^{2}}{8\,\varepsilon_{m'}}-\varepsilon_{k'}\right)\cdot i\cdot 10^{-3m/4}+[10^{-m}]\Big\}$$

$$\sqrt{v_2^2-4(u_2-w_2)}=\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}\cdot 10^{\beta-m/4}+\frac{\varepsilon_{l'}}{2\cdot\sqrt{\varepsilon_{m'}}}\cdot i\cdot 10^{\beta-m/2}$$

$$+\frac{1}{2\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}}\left(\frac{\varepsilon_{l'}^{2}}{8\,\varepsilon_{m'}}-\varepsilon_{k'}\right)\cdot 10^{\beta-3m/4}+[10^{\beta-5m/4}]$$

$$\left\{\begin{aligned}
y_3&=\frac{1}{2}\left\{v_2+\sqrt{v_2^2-4(u_2-w_2)}\right\}=\frac{(1+i)}{\sqrt{2}}\cdot\sqrt[4]{\varepsilon_{m'}}\cdot 10^{\beta-m/4}\\
&\quad+\frac{\varepsilon_{l'}}{4\cdot\sqrt{\varepsilon_{m'}}}\cdot i\cdot 10^{\beta-m/2}+\frac{(1-i)}{4\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}}\left(\frac{\varepsilon_{l'}^{2}}{8\,\varepsilon_{m'}}-\varepsilon_{k'}\right)\cdot 10^{\beta-3m/4}+[10^{\beta-5m/4}]\\
y_1&=\frac{1}{2}\left\{v_2-\sqrt{v_2^2-4(u_2-w_2)}\right\}=-\frac{(1-i)}{\sqrt{2}}\cdot\sqrt[4]{\varepsilon_{m'}}\cdot 10^{\beta-m/4}\\
&\quad-\frac{\varepsilon_{l'}}{4\cdot\sqrt{\varepsilon_{m'}}}\,i\cdot 10^{\beta-m/2}-\frac{(1+i)}{4\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}}\left(\frac{\varepsilon_{l'}^{2}}{8\,\varepsilon_{m'}}-\varepsilon_{k'}\right)\cdot 10^{\beta-3m/4}+[10^{\beta-5m/4}]
\end{aligned}\right.\tag{21}$$

$u_3$ (9) を用いて (2) を因数分解した2つの2次方程式 (24), (25) の係数を求める。

$$2u_3-k'=-\frac{\varepsilon_{l'}^{2}}{4\,\varepsilon_{m'}}\cdot 10^{2\beta-m}+[10^{2\beta-2m}]$$

$$v_3=\sqrt{2u_3-k'}=\frac{\varepsilon_{l'}}{2\cdot\sqrt{\varepsilon_{m'}}}\cdot i\cdot 10^{\beta-m/2}+[10^{\beta-3m/2}]\tag{22}$$

$$u_3^2=\left(\frac{\varepsilon_{k'}}{2}-\frac{\varepsilon_{l'}^{2}}{8\,\varepsilon_{m'}}\right)^2\cdot 10^{4\beta-2m}+[10^{4\beta-3m}]$$

$$w_3=\sqrt{u_3^2-m'}=\sqrt{\varepsilon_{m'}}\cdot i\cdot 10^{2\beta-m/2}+[10^{2\beta-3m/2}]\tag{23}$$

$$y^2+v_3y+(u_3+w_3)=0\tag{24}$$

$$y^2-v_3y+(u_3-w_3)=0\tag{25}$$

2次方程式(24)より(2)の解 $y_1$, $y_4$ を求める.

$$v_3=\frac{\varepsilon_{l'}}{2\cdot\sqrt{\varepsilon_{m'}}}\cdot i\cdot 10^{\beta-m/2}+[10^{\beta-3m/2}]$$

$$u_3+w_3=\sqrt{\varepsilon_{m'}}\cdot i\cdot 10^{2\beta-m/2}+\left(\frac{\varepsilon_{k'}}{2}-\frac{\varepsilon_{l'}^2}{8\varepsilon_{m'}}\right)\cdot 10^{2\beta-m}+[10^{2\beta-3m/2}]$$

$$v_3^2=\qquad -\frac{\varepsilon_{l'}^2}{4\varepsilon_{m'}}\cdot 10^{2\beta-m}+[10^{2\beta-2m}]$$

$$-4(u_3+w_3)=-4\sqrt{\varepsilon_{m'}}\cdot i\cdot 10^{2\beta-m/2}-4\left(\frac{\varepsilon_{k'}}{2}-\frac{\varepsilon_{l'}^2}{8\varepsilon_{m'}}\right)\cdot 10^{2\beta-m}+[10^{2\beta-3m/2}]$$

$$v_3^2-4(u_3+w_3)=-4\sqrt{\varepsilon_{m'}}\cdot i\cdot 10^{2\beta-m/2}\left\{1+\frac{1}{2\sqrt{\varepsilon_{m'}}}\left(\frac{\varepsilon_{l'}^2}{8\varepsilon_{m'}}-\varepsilon_{k'}\right)\cdot i\cdot 10^{-m/2}+[10^{-m}]\right\}$$

$$\sqrt{v_3^2-4(u_3+w_3)}=-\sqrt{2}\cdot(1-i)\cdot\sqrt[4]{\varepsilon_{m'}}\cdot 10^{\beta-m/4}-\frac{(1+i)}{2\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}}\left(\frac{\varepsilon_{l'}^2}{8\varepsilon_{m'}}-\varepsilon_{k'}\right)\cdot 10^{\beta-3m/4}+[10^{\beta-5m/4}]$$

$$\begin{cases} y_1=\frac{1}{2}\left\{-v_3+\sqrt{v_3^2-4(u_3+w_3)}\right\}=-\frac{(1-i)}{\sqrt{2}}\cdot\sqrt[4]{\varepsilon_{m'}}\cdot 10^{\beta-m/4} \\ \qquad -\frac{\varepsilon_{l'}}{4\cdot\sqrt{\varepsilon_{m'}}}\cdot i\cdot 10^{\beta-m/2}-\frac{(1+i)}{4\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}}\left(\frac{\varepsilon_{l'}^2}{8\varepsilon_{m'}}-\varepsilon_{k'}\right)\cdot 10^{\beta-3m/4}+[10^{\beta-5m/4}] \\ y_4=\frac{1}{2}\left\{-v_3-\sqrt{v_3^2-4(u_3+w_3)}\right\}=\frac{(1-i)}{\sqrt{2}}\cdot\sqrt[4]{\varepsilon_{m'}}\cdot 10^{\beta-m/4} \\ \qquad -\frac{\varepsilon_{l'}}{4\cdot\sqrt{\varepsilon_{m'}}}\cdot i\cdot 10^{\beta-m/2}+\frac{(1+i)}{4\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}}\left(\frac{\varepsilon_{l'}^2}{8\varepsilon_{m'}}-\varepsilon_{k'}\right)\cdot 10^{\beta-3m/4}+[10^{\beta-5m/4}] \end{cases} \tag{26}$$

2次方程式(25)より(2)の解 $y_3$, $y_2$ を求める.

$$u_3 - w_3 = -\sqrt{\varepsilon_{m'}}\cdot i\cdot 10^{2\beta-m/2} + \left(\frac{\varepsilon_{k'}}{2} - \frac{\varepsilon_{l'}^2}{8\,\varepsilon_{m'}}\right)\cdot 10^{2\beta-m} + [\,10^{2\beta-3m/2}\,]$$

$$-4(u_3 - w_3) = 4\sqrt{\varepsilon_{m'}}\cdot i\cdot 10^{2\beta-m/2} - 4\left(\frac{\varepsilon_{k'}}{2} - \frac{\varepsilon_{l'}^2}{8\,\varepsilon_{m'}}\right)\cdot 10^{2\beta-m} + [\,10^{2\beta-3m/2}\,]$$

$$v_3^2 - 4(u_3 - w_3) = 4\sqrt{\varepsilon_{m'}}\cdot i\cdot 10^{2\beta-m/2}\left\{1 - \frac{1}{2\cdot\sqrt{\varepsilon_{m'}}}\left(\frac{\varepsilon_{l'}^2}{8\,\varepsilon_{m'}} - \varepsilon_{k'}\right)\cdot i\cdot 10^{-m/2} + [\,10^{-m}\,]\right\}$$

$$\sqrt{v_3^2 - 4(u_3 - w_3)} = \sqrt{2}\cdot(1+i)\cdot\sqrt[4]{\varepsilon_{m'}}\cdot 10^{\beta-m/4} + \frac{(1-i)}{2\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}}\left(\frac{\varepsilon_{l'}^2}{8\,\varepsilon_{m'}} - \varepsilon_{k'}\right)\cdot 10^{\beta-3m/4} + [\,10^{\beta-5m/4}\,]$$

$$\left\{\begin{aligned} y_3 &= \frac{1}{2}\left\{v_3 + \sqrt{v_3^2 - 4(u_3 - w_3)}\right\} = \frac{(1+i)}{\sqrt{2}}\cdot\sqrt[4]{\varepsilon_{m'}}\cdot 10^{\beta-m/4} \\ &\quad + \frac{\varepsilon_{l}}{4\cdot\sqrt{\varepsilon_{m'}}}\cdot i\cdot 10^{\beta-m/2} + \frac{(1-i)}{4\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}}\left(\frac{\varepsilon_{l'}^2}{8\,\varepsilon_{m'}} - \varepsilon_{k'}\right)\cdot 10^{\beta-3m/4} + [\,10^{\beta-5m/4}\,] \\ y_2 &= \frac{1}{2}\left\{v_3 - \sqrt{v_3^2 - 4(u_3 - w_3)}\right\} = -\frac{(1+i)}{\sqrt{2}}\cdot\sqrt[4]{\varepsilon_{m'}}\cdot 10^{\beta-m/4} \\ &\quad + \frac{\varepsilon_{l'}}{4\cdot\sqrt{\varepsilon_{m'}}}\cdot i\cdot 10^{\beta-m/2} - \frac{(1-i)}{4\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}}\left(\frac{\varepsilon_{l'}^2}{8\,\varepsilon_{m'}} - \varepsilon_{k'}\right)\cdot 10^{\beta-3m/4} + [\,10^{\beta-5m/4}\,] \end{aligned}\right. \tag{27}$$

3次方程式(5)の解$u_1$, $u_2$, $u_3$ (9)のいずれを用いても．当然のことながら．4次方程式(2)の解$y_1$, $y_2$, $y_3$, $y_4$は等しい値が得られる．したがって．4次方程式(1)の解は(28)である。

$$
\left\{
\begin{aligned}
x_1 = y_1 + \bar{x} &= \alpha\cdot 10^{\beta} - \frac{(1-i)}{\sqrt{2}}\cdot\sqrt[4]{\varepsilon_{m'}}\cdot 10^{\beta-m/4} - \frac{\varepsilon_{\ell'}}{4\cdot\sqrt{\varepsilon_{m'}}}\cdot i\cdot 10^{\beta-m/2} \\
&\quad - \frac{(1+i)}{4\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}}\left(\frac{\varepsilon_{\ell'}^2}{8\varepsilon_{m'}} - \varepsilon_{k'}\right)\cdot 10^{\beta-3m/4} + [10^{\beta-m}] \\
x_2 = y_2 + \bar{x} &= \alpha\cdot 10^{\beta} - \frac{(1+i)}{\sqrt{2}}\cdot\sqrt[4]{\varepsilon_{m'}}\cdot 10^{\beta-m/4} + \frac{\varepsilon_{\ell'}}{4\cdot\sqrt{\varepsilon_{m'}}}\cdot i\cdot 10^{\beta-m/2} \\
&\quad - \frac{(1-i)}{4\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}}\left(\frac{\varepsilon_{\ell'}^2}{8\varepsilon_{m'}} - \varepsilon_{k'}\right)\cdot 10^{\beta-3m/4} + [10^{\beta-m}] \\
x_3 = y_3 + \bar{x} &= \alpha\cdot 10^{\beta} + \frac{(1+i)}{\sqrt{2}}\cdot\sqrt[4]{\varepsilon_{m'}}\cdot 10^{\beta-m/4} + \frac{\varepsilon_{\ell'}}{4\cdot\sqrt{\varepsilon_{m'}}}\cdot i\cdot 10^{\beta-m/2} \\
&\quad + \frac{(1-i)}{4\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}}\left(\frac{\varepsilon_{\ell'}^2}{8\varepsilon_{m'}} - \varepsilon_{k'}\right)\cdot 10^{\beta-3m/4} + [10^{\beta-m}] \\
x_4 = y_4 + \bar{x} &= \alpha\cdot 10^{\beta} + \frac{(1-i)}{\sqrt{2}}\cdot\sqrt[4]{\varepsilon_{m'}}\cdot 10^{\beta-m/4} - \frac{\varepsilon_{\ell'}}{4\cdot\sqrt{\varepsilon_{m'}}}\cdot i\cdot 10^{\beta-m/2} \\
&\quad + \frac{(1+i)}{4\cdot\sqrt{2}\cdot\sqrt[4]{\varepsilon_{m'}}}\left(\frac{\varepsilon_{\ell'}^2}{8\varepsilon_{m'}} - \varepsilon_{k'}\right)\cdot 10^{\beta-3m/4} + [10^{\beta-m}]
\end{aligned}
\right.
\tag{28}
$$

解と係数の関係を用いて、解より係数を求める。

$$\underline{y_1 + y_2 + y_3 + y_4} = \bar{\varepsilon}\cdot 10^{\beta-5m/4} + [10^{\beta-9m/4}] \quad \{= 0.\}$$

$$y_1\cdot y_4 = \sqrt{\varepsilon_{m'}}\cdot i\cdot 10^{2\beta-m/2} - \frac{1}{2}\left(\frac{\varepsilon_{\ell'}^2}{4\varepsilon_{m'}} - \varepsilon_{k'}\right)\cdot 10^{2\beta-m} + [10^{2\beta-3m/2}]$$

$$y_2\cdot y_3 = -\sqrt{\varepsilon_{m'}}\cdot i\cdot 10^{2\beta-m/2} - \frac{1}{2}\left(\frac{\varepsilon_{\ell'}^2}{4\varepsilon_{m'}} - \varepsilon_{k'}\right)\cdot 10^{2\beta-m} + [10^{2\beta-3m/2}]$$

$$y_1 + y_4 = -\frac{\varepsilon_{\ell'}}{2\cdot\sqrt{\varepsilon_{m'}}}\cdot i\cdot 10^{\beta-m/2} + \delta_1\cdot 10^{\beta-5m/4} + [10^{\beta-3m/2}]$$

$$y_2 + y_3 = \frac{\varepsilon_{\ell'}}{2\cdot\sqrt{\varepsilon_{m'}}}\cdot i\cdot 10^{\beta-m/2} + \delta_2\cdot 10^{\beta-5m/4} + [10^{\beta-3m/2}]$$

$$\underline{y_1\cdot y_4 + y_2\cdot y_3 + y_1\cdot y_2 + y_1\cdot y_3 + y_4\cdot y_2 + y_4\cdot y_3}$$

$$= y_1\cdot y_4 + y_2\cdot y_3 + (y_1 + y_4)(y_2 + y_3)$$

$$= \varepsilon_{k'}\cdot 10^{2\beta-m} + \delta_3\cdot 10^{2\beta-3m/2} + \frac{\varepsilon_{\ell'}}{2\cdot\sqrt{\varepsilon_{m'}}}(\delta_1 - \delta_2)\cdot 10^{2\beta-7m/4} + [10^{2\beta-2m}]\ \{= k'\}$$

$$y_1\cdot y_4\cdot(y_2 + y_3) = -\frac{\varepsilon_{\ell'}}{2}\cdot 10^{3\beta-m} - \frac{\varepsilon_{\ell}}{4\cdot\sqrt{\varepsilon_{m'}}}\left(\frac{\varepsilon_{\ell'}^2}{4\,\varepsilon_{m}} - \varepsilon_{k'}\right)\cdot i\cdot 10^{3\beta-3m/2}$$

$$+\delta_2\cdot\sqrt{\varepsilon_{m'}}\cdot i\cdot 10^{3\beta-7m/4} + [10^{3\beta-2m}]$$

$$y_2\cdot y_3\cdot(y_1 + y_4) = -\frac{\varepsilon_{\ell'}}{2}\cdot 10^{3\beta-m} + \frac{\varepsilon_{\ell'}}{4\cdot\sqrt{\varepsilon_{m'}}}\left(\frac{\varepsilon_{\ell'}^2}{4\,\varepsilon_{m'}} - \varepsilon_{k'}\right)\cdot i\cdot 10^{3\beta-3m/2}$$

$$-\delta_1\sqrt{\varepsilon_{m'}}\cdot i\cdot 10^{3\beta-7m/4} + [10^{3\beta-2m}]$$

$$\underline{y_1\cdot y_2\cdot y_3 + y_1\cdot y_2\cdot y_4 + y_1\cdot y_3\cdot y_4 + y_2\cdot y_3\cdot y_4}$$

$$= y_1\cdot y_4\cdot(y_2 + y_3) + y_2\cdot y_3\cdot(y_1 + y_4)$$

$$= -\varepsilon_{\ell'}\cdot 10^{3\beta-m} + (\delta_2 - \delta_1)\cdot\sqrt{\varepsilon_{m'}}\cdot i\cdot 10^{3\beta-7m/4} + [10^{3\beta-2m}]\ \{= -\ell'\}$$

$$\underline{y_1\cdot y_2\cdot y_3\cdot y_4} = \varepsilon_{m'}\cdot 10^{4\beta-m} + [10^{4\beta-2m}]\ \{= m'\}$$

$$x_1 + x_2 + x_3 + x_4 = 4\alpha\cdot 10^{\beta} + [10^{\beta-m}]\ \{= -a_3\}$$

$$x_1\cdot x_4 = \alpha^2\cdot 10^{2\beta} + \left(\sqrt{\varepsilon_m} - \frac{\alpha\cdot\varepsilon_{\ell}}{2\cdot\sqrt{\varepsilon_{m'}}}\right)\cdot i\cdot 10^{2\beta-m/2} + [10^{2\beta-m}]$$

$$x_2\cdot x_3 = \alpha^2\cdot 10^{2\beta} - \left(\sqrt{\varepsilon_m} - \frac{\alpha\cdot\varepsilon_{\ell}}{2\cdot\sqrt{\varepsilon_{m}}}\right)\cdot i\cdot 10^{2\beta-m/2} + [10^{2\beta-m}]$$

$$x_1 + x_4 = 2\alpha\cdot 10^{\beta} - \frac{\varepsilon_{\ell'}}{2\cdot\sqrt{\varepsilon_{m'}}}\cdot i\cdot 10^{\beta-m/2} + [10^{\beta-m}]$$

$$x_2+x_3=2\alpha\cdot10^{\beta}+\frac{\varepsilon_{\ell'}}{2\cdot\sqrt{\varepsilon_{m'}}}\cdot i\cdot10^{\beta-m/2}+[10^{\beta-m}]$$

$$\underline{x_1\cdot x_4+x_2\cdot x_3+x_1\cdot x_2+x_1\cdot x_3+x_4\cdot x_2+x_4\cdot x_3}$$

$$=x_1\cdot x_4+x_2\cdot x_3+(x_1+x_4)(x_2+x_3)$$

$$=6\alpha^2\cdot10^{2\beta}+[10^{2\beta-m}]\ \{=a_2\}$$

$$x_1\cdot x_4\cdot(x_2+x_3)$$

$$=2\alpha^3\cdot10^{3\beta}+\left\{2\alpha\cdot\sqrt{\varepsilon_{m'}}-\frac{\alpha^2\cdot\varepsilon_{\ell'}}{2\cdot\sqrt{\varepsilon_{m'}}}\right\}\cdot i\cdot10^{3\beta-m/2}+[10^{3\beta-m}]$$

$$x_2\cdot x_3\cdot(x_1+x_4)$$

$$=2\alpha^3\cdot10^{3\beta}-\left\{2\alpha\cdot\sqrt{\varepsilon_{m'}}-\frac{\alpha^2\cdot\varepsilon_{\ell'}}{2\cdot\sqrt{\varepsilon_{m'}}}\right\}\cdot i\cdot10^{3\beta-m/2}+[10^{3\beta-m}]$$

$$\underline{x_1\cdot x_2\cdot x_3+x_1\cdot x_2\cdot x_4+x_1\cdot x_3\cdot x_4+x_2\cdot x_3\cdot x_4}$$

$$=x_1\cdot x_4\cdot(x_2+x_3)+x_2\cdot x_3\cdot(x_1+x_4)$$

$$=4\alpha^3\cdot10^{3\beta}+[10^{3\beta-m}]\ \{=-a_1\}$$

$$\underline{x_1\cdot x_2\cdot x_3\cdot x_4}=\alpha^4\cdot10^{4\beta}+[10^{4\beta-m}]\ \{=a_0\}$$

$\varepsilon$, $\delta_1$, $\delta_2$, $\delta_3$ ： 誤差のみの数値の数値部

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